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Finiteness of $\mathcal{N}=4$ super-Yang–Mills effective action in terms of dressed $\mathcal{N}=1$ superfields

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Abstract

We argue in favor of the independence on any scale, ultraviolet or infrared, in kernels of the effective action expressed in terms of dressed $\mathcal{N}=1$ superfields for the case of $\mathcal{N}=4$ super-Yang–Mills theory. Under "finiteness" of the effective action of dressed mean superfields we mean its "scale independence". We use two types of regularization: regularization by dimensional reduction and regularization by higher derivatives in its supersymmetric form. Based on the Slavnov–Taylor identity we show that dressed fields of matter and of vector multiplets can be introduced to express the effective action in terms of them. Kernels of the effective action expressed in terms of such dressed effective fields do not depend on the ultraviolet scale. In the case of dimensional reduction, by using the developed technique we show how the problem of inconsistency of the dimensional reduction can be solved. Using Piguet and Sibold formalism, we indicate that the dependence on the infrared scale disappears off shell in both the regularizations.

Keywords: R-operation, gauge symmetry, $\mathcal{N}=4$ supersymmetry, Slavnov–Taylor identity.

The effective action is restricted by consequences of various symmetries of the classical action that at the quantum level take the form of specific identities. One of them is the Slavnov–Taylor (ST) identity [1]-[6]. This generalizes the Ward–Takahashi identity of quantum electrodynamics to the non-Abelian case and can be derived starting from the property of invariance of the tree-level action with respect to BRST symmetry [7, 8]. The ST identity can be formulated as equations involving variational derivatives of the effective action. In the general $\mathcal{N}=1$ supersymmetric theory it can be written as [9]

$$\begin{aligned}
&\mathrm{Tr}\Big[\int d^8z\ \frac{\delta\Gamma}{\delta V}\frac{\delta\Gamma}{\delta K} - i\int d^6y\ \frac{\delta\Gamma}{\delta c}\frac{\delta\Gamma}{\delta L} + i\int d^6\bar{y}\ \frac{\delta\Gamma}{\delta\bar{c}}\frac{\delta\Gamma}{\delta\bar{L}} \\
&- \int d^6y\ \frac{\delta\Gamma}{\delta b}\left(\frac{1}{32}\frac{1}{\alpha}\bar{D}^2D^2V\right) - \int d^6\bar{y}\ \frac{\delta\Gamma}{\delta\bar{b}}\left(\frac{1}{32}\frac{1}{\alpha}D^2\bar{D}^2V\right)\Big] \\
&-i\int d^6y\ \frac{\delta\Gamma}{\delta\Phi}\ \frac{\delta\Gamma}{\delta k} + i\int d^6\bar{y}\ \frac{\delta\Gamma}{\delta\bar{k}}\ \frac{\delta\Gamma}{\delta\bar{\Phi}} = 0.
\end{aligned}$$

Here the standard definition of the measures in superspace is used

$$d^8z \equiv d^4x\ d^2\theta\ d^2\bar{\theta}, \quad d^6y \equiv d^4y\ d^2\theta, \quad d^6\bar{y} \equiv d^4\bar{y}\ d^2\bar{\theta}.$$

The effective action Γ generates one particle irreducible amplitudes of the quantum fields and contains all the information about the quantum behaviour of the theory. It is a functional of the effective fields $V, b, \bar{b}, c, \bar{c}, \Phi, \bar{\Phi}$ and external sources $K, L, \bar{L}, k, \bar{k}$, coupled at tree level to BRST-transformations of the corresponding classical fields [4]. We use two types of regularization: regularization by higher derivatives [10, 11] and regularization by dimensional reduction [12, 13].

We will show that the actual variables of the effective action are dressed effective superfields, that is, they are effective superfields convoluted with some unspecified dressing functions that are parts of propagators. Kernels accompanying the dressed effective fields in the effective action are related to the scattering amplitudes of the particles. A similar problem has been solved in component formalism [14, 15]. As has been argued in Refs. [14, 15] in that formalism the dressed mean fields appear to be the actual variables of the effective action, leaving the kernels of the action independent of any scale, ultraviolet or infrared, in case of $\mathcal{N}=4$ supersymmetric Yang–Mills theory. This statement has been confirmed by the explicit calculation in Ref. [16]. In the present paper this problem is considered by using $\mathcal{N}=1$ superfield formalism which keep one of the supersymmetries apparent and conserve all the R-symmetries which we need to apply anomaly multiplet ideas [17]. The important point of Refs. [14, 15, 18, 19, 20, 22, 21, 23] is the possibility to absorb the two point proper functions in the re-definition of the effective fields.

We consider the $\mathcal{N}=4$ theory in $\mathcal{N}=1$ superfield formalism. This model has specific field contents. The Lagrangian of the model in terms of $\mathcal{N}=1$ superfields is

$$\begin{aligned}
S = \frac{1}{g^2}\frac{1}{128}\mathrm{Tr}\ \Big[&\int d^6y\ W_\alpha W^\alpha + \int d^6\bar{y}\ \bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}} \\
&+\int d^8z\ e^{-V}\bar{\Phi}_i\ e^V\ \Phi^i \\
&+\frac{1}{3!}\int d^6y\ i\epsilon_{ijk}\Phi^i[\Phi^j,\Phi^k] + \frac{1}{3!}\int d^6\bar{y}\ i\epsilon^{ijk}\bar{\Phi}_i[\bar{\Phi}_j,\bar{\Phi}_k]\Big].
\end{aligned}$$

For the $\mathcal{N}=1$ supersymmetry we use the notation of Ref. [24]. This Lagrangian for $\mathcal{N}=4$ supersymmetry is taken from Ref. [25]. The flavor indices of the matter run in $i=1,2,3$ and the matter superfields are in the adjoint representation of the gauge group, $\Phi^i = \Phi^{ia}\ T^a$. (The vector superfield is in the same representation, $V = V^a T^a$).

Consider for the beginning the general $\mathcal{N}=1$ super-Yang–Mills whose classical action takes the form

$$\begin{aligned} S = \int\ d^6y\ \frac{1}{128}\frac{1}{g^2}\mathrm{Tr}\ W_\alpha W^\alpha + \int\ d^6\bar{y}\ \frac{1}{128}\frac{1}{g^2}\mathrm{Tr}\ \bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}} \\ + \int\ d^8z\ \bar{\Phi}\ e^V\ \Phi \\ + \int\ d^6y\ \Big[Y^{ijk}\Phi_i\Phi_j\Phi_k + M^{ij}\Phi_i\Phi_j\Big] + \int\ d^6\bar{y}\ \Big[\bar{Y}_{ijk}\bar{\Phi}^i\bar{\Phi}^j\bar{\Phi}^k + \bar{M}_{ij}\bar{\Phi}^i\bar{\Phi}^j\Big]. \end{aligned} \tag{1}$$

We do not specify the representation of the matter fields here. It is some general reducible representation of the gauge group with a set of irreducible representations. The Yukawa couplings Y^{ijk} and M^{ij} appear at some general triple vertex and mass terms in four dimensions. The path integral describing the quantum theory is defined as

$$\begin{aligned} Z[J,\eta,\bar{\eta},\rho,\bar{\rho},j,\bar{j},K,L,\bar{L},k,\bar{k}] = \int\ dV\ dc\ d\bar{c}\ db\ d\bar{b}\ d\Phi\ d\bar{\Phi}\ \exp i\,[S \\ +2\ \mathrm{Tr}\left(\int\ d^8z\ JV + i\int\ d^6y\ \eta c + i\int\ d^6\bar{y}\ \bar{\eta}\bar{c} + i\int\ d^6y\ \rho b + i\int\ d^6\bar{y}\ \bar{\rho}\bar{b}\right) \\ + \left(\int\ d^6y\ \Phi\ j + \int\ d^6\bar{y}\ \bar{j}\ \bar{\Phi}\right) \\ +2\ \mathrm{Tr}\left(i\int\ d^8z\ K\delta_{\bar{c},c}V + \int\ d^6y\ Lc^2 + \int\ d^6\bar{y}\ \bar{L}\bar{c}^2\right) \\ + \int\ d^6y\ k\ c\ \Phi + \int\ d^6\bar{y}\ \bar{\Phi}\ \bar{c}\ \bar{k}\Big], \end{aligned} \tag{2}$$

The derivation of the ST identity in general supersymmetric theory can be found for example in Ref. [23]. The result is

$$\begin{aligned} \mathrm{Tr}\Big[\int\ d^8z\ \frac{\delta\Gamma}{\delta V}\frac{\delta\Gamma}{\delta K} - i\int\ d^6y\ \frac{\delta\Gamma}{\delta c}\frac{\delta\Gamma}{\delta L} + i\int\ d^6\bar{y}\ \frac{\delta\Gamma}{\delta\bar{c}}\frac{\delta\Gamma}{\delta\bar{L}} \\ - \int\ d^6y\ \frac{\delta\Gamma}{\delta b}\left(\frac{1}{32}\frac{1}{\alpha}\bar{D}^2D^2V\right) - \int\ d^6\bar{y}\ \frac{\delta\Gamma}{\delta\bar{b}}\left(\frac{1}{32}\frac{1}{\alpha}D^2\bar{D}^2V\right)\Big] \\ -i\ \int\ d^6y\ \frac{\delta\Gamma}{\delta\Phi}\ \frac{\delta\Gamma}{\delta k} + i\ \int\ d^6\bar{y}\ \frac{\delta\Gamma}{\delta\bar{k}}\ \frac{\delta\Gamma}{\delta\bar{\Phi}} = 0. \end{aligned} \tag{3}$$

Regularization is necessary to analyze the identities. First, we consider the regularization by dimensional reduction. Under this regularization the algebra of Lorentz indices is done in four dimensions but integration is done in $4-2\epsilon$ dimensions in the momentum or in the position space. As has been shown in Refs. [14, 15], such a regularization is self-consistent at least in $\mathcal{N}=4$ supersymmetric Yang–Mills theory in component formalism at all orders of the perturbation theory. In the next paragraphs we will show such a regularization procedure can be applied at *all* orders of the perturbation theory in superfield

formalism too if this regularization is combined with Piguet and Sibold formalism of Refs. [26, 27]

Let's analyse this identity in the following way. One can start by considering the monomial Lcc of the effective action. Due to supersymmetry, superficial divergences are absent in chiral vertices [28, 29]. This theorem is a direct consequence of the Grassmanian integration and has been described, e.g., in Ref. [28]. However, there could be finite contributions. They are scale-independent, in that sense they remain finite in the limit of removing the ultraviolet regularization. For example, at one loop level one can find among others the following kernel structure for the correlator Lcc [14]:

$$\int d^4\theta \; d^4x_1 d^4x_2 d^4x_3 \; \frac{1}{(x_1-x_2)^2(x_1-x_3)^2(x_2-x_3)^4} f^{bca} \times$$
$$\times \left(D^2 \; L^a(x_1,\theta,\bar\theta)\right) c^b(x_2,\theta,\bar\theta) c^c(x_3,\theta,\bar\theta).$$

Landau gauge is the specific case in gauge theories because we do not need to renormalize the gauge fixing parameter. Absence of the gauge parameter is enough condition to avoid this possible source of appearance of the scale dependence in kernels of dressed mean superfields through the renormalization of the gauge parameter. From the effective action Γ we can extract the two point ghost proper correlator $G(z-z')$,

$$G^\dagger = G,$$

and a two point connected ghost correlator $G^{-1}(z-z')$, which is related to the previous one in the following way:

$$\int d^8z' \; G(z_1-z') \; G^{-1}(z_2-z') = \delta^{(8)}(z_2-z_1).$$

This definition is valid in each order of perturbation theory. One can absorb this two point proper function into a non-local redefinition of the effective fields K and V in the following manner [20]:

$$\begin{aligned} \tilde V &\equiv \int \; d^8z' \; V(z') \; G^{-1}(z-z'), \\ \tilde K &\equiv \int \; d^8z' \; K(z') \; G(z-z'). \end{aligned} \tag{4}$$

One can see that the part of the ST identity without the gauge fixing term is covariant with respect to such a re-definition of the effective fields. We will call the construction (4) dressed effective (or mean) superfields. Proceeding at one loop level in terms of the dressed effective superfields, one can see from the ST identity that the divergence of the $\tilde K c \tilde V$ vertex must be canceled by the divergence of the Lcc vertex. However, the $\mathcal{N}=1$ Lcc vertex does not diverge at one loop level in the momentum space due to supersymmetry.[1] This means that the ST identity clearly shows that the $\tilde K c \tilde V$ is also finite [14, 15], that is, it does not diverge in the limit of removing the regularization. The

[1]For simplicity we consider the Landau gauge.

rest of the UV divergence in the propagator of the dressed gauge fields $\tilde{V}\tilde{V}$ can be removed by redefining of the gauge coupling constant. In theories with zero beta function, in case of this paper it is $\mathcal{N}=4$ supersymmetric Yang-Mills theory, this last divergence is absent. The other graphs are finite since proper correlators can be constructed from the $\tilde{K}c\tilde{V}$ (or Lcc), and $\tilde{V}\tilde{V}$ correlators by means of the ST identity. This is a direct consequence of the ST identity and R-operation [14, 33]. We can repeat this argument in each order of perturbation theory, which is completely the same as in Refs. [14, 15] in component formalism.

Thus, all the gauge part of the effective action can be considered as the functional of the effective fields $\tilde{V}$, $\tilde{K}$, L, c, $\bar{c}$, $\tilde{b}$ and $\tilde{\bar{b}}$. Due to the antighost equation [20], the antighost field b is always dressed in the same manner as the auxiliary field K is dressed. Kernels of this effective action are functions of the gauge coupling, mutual distances and in general of the ultraviolet scale, because the divergences in subgraphs must be removed by the renormalization of the gauge coupling. But if the beta function is zero the kernels have no UV scale dependence. In the position space in component formalism the infrared divergences can be analysed in the same way like ultraviolet divergences were analysed in Ref.[33] in the momentum space by means of $R-$operation [16]. However, it is difficult to repeat this argument in the position space in superfield formalism since the propagator of vector superfield is dangerous in the infrared region. In view of this difficulty we use the formalism developed in Refs. [26, 27], where the problem of off-shell infrared divergences of superfield formalism has been solved.

The infrared regulator has been introduced by means of the following trick of renormalization of the vector gauge field V :

$$V \rightarrow V + \mu\theta^2\bar{\theta}^2 V,$$

where μ is the infrared regulator mass. Propagators of the lowest components of the gauge superfield obtain a shift by the infrared regulator mass which is enough to make the Feynman graphs safe in the infrared region of momentum space. It appears one can construct the classical action that satisfies a generalized ST identity which involves BRST counterparts of the gauge parameters. Then, a general solution to the generalized ST identity has been found at the classical level. As a consequence of that solution, the path integral that corresponds to this solution possesses the property of independence of v.e.v.s of gauge invariant quantities on the gauge parameters [27]. The independence of the physical quantities on the infrared scale was obtained by the same way. In addition, the new external superfield u can be introduced so that a shift of its highest component is proportional to the infrared scale μ. That field also participates in the generalized ST identity.

One can analyse the generalized ST identity of Ref. [27] which is obtained after the modification of standard one (3) by including additional external fields and gauge parameters. Appearance of the additional insertions of the external field u or spurions $\mu\theta^2\bar{\theta}^2$ into supergraphs does not change the nonrenormalization theorems. This property has been used in Refs. [30, 31, 32] to derive the relation between softly broken and rigid renormalization constants in $\mathcal{N}=1$ supersymmetric theory. Thus, it cannot bring any changes for

our conclusions about the Lcc vertex from the point of our analysis since all divergent subgraphs remain divergent and all convergence properties of subgraphs remain unchanged. In that sense subgraphs are finite after renormalization by the dressing functions but all the vertex as a whole is also finite superficially due to property of Grassmanian integration which is not broken by the insertions of the external superfields [30, 31, 32].

There is also another way to explain the independence of the physical quantities on the infrared scale μ. The point is that the the factors $(1 + \mu\theta^2\bar{\theta}^2)$ coming from vertices will be canceled with factors $(1 + \mu\theta^2\bar{\theta}^2)^{-1}$ coming from propagators. However, the propagators are IR-finite with the μ addition and thus the theory is regularized in the infrared. The same trick can be applied to demonstrate the independence of the correlators of dressed mean superfields in the Landau gauge of the infrared scale μ.

In this paper we have significantly used the vanishing of the gauge beta function in $\mathcal{N} = 4$ super-Yang-Mills theory. The vanishing of the beta function in first three orders of the perturbation theory has been established in Refs. [34, 35, 36]. Originally, in the background field technique, it has been shown in Ref. [37] that in $\mathcal{N} = 2$ supersymmetric Yang–Mills theory the beta function vanishes beyond one loop. The same result has been derived in Ref. [38] by using the background field technique with unconstrained $\mathcal{N} = 2$ superfields. The arguments of [37, 38] are based on the fact that $\mathcal{N} = 2$ supersymmetry prohibits any counterterms to the gauge coupling except for one loop contribution. In Ref. [29], the fact that $\mathcal{N} = 2$ supersymmetric YM theory does not have contributions to the beta function beyond one loop has been argued based on currents of R-symmetry, which are in the same supermultiplet with the energy-momentum tensor. The proportionality of the trace anomaly of the energy momentum tensor to the beta function in general nonsupersymmetric Yang-Mills theories has been proved in Ref. [39]. Since R-symmetry does not have anomaly in $\mathcal{N} = 4$ theory, the same is true for the anomaly of the conformal symmetry which is proportional to the beta function. At one loop level the beta function is zero with this field contents [34, 35]. Moreover, explicit calculation has been done at two loops in terms of $\mathcal{N} = 1$ superfields [40], and it has been shown that the beta function of $\mathcal{N} = 2$ theory is zero at two loops.

The dimensional reduction was known to be inconsistent [41]. We proposed solution to this problem in component formalism in Ref. [15] for $\mathcal{N} = 4$ super-Yang-Mills theory. We can repeat similar arguments in case of superfield formalism. The only new feature here is the appearance of the infrared scale μ in subgraphs, as it has been explained above. The point is that the vertex Lcc is always convergent superficially in superfield formalism. In $\mathcal{N} = 4$ super-Yang-Mills theory ultraviolet divergences in the subgraphs of the Lcc vertex should cancel each other at the end. The insertion of the operator of the conformal anomaly into vacuum expectation values of operators of gluonic fields at different points in spacetime is proportional to the beta function of the gauge coupling [39]. Due to the algebra of the four-dimensional supersymmetry the beta function should be zero [17]. Algebra of the supersymmetry operators in the Hilbert space created by dressed fields can be considered as four-dimensional in Lorentz indices as well as in spinor indices since the limit $\epsilon \to 0$ is non-singular at one-loop order, two-loop order and higher orders as we have seen in the previous paragraphs. Thus, we can consider each correlator as pure four-dimensional, solving order-by-order the problem of dimensional discrepancy

of convolutions in Lorentz and spinor indices. The dependence on the infrared scale μ is canceled by itself from the contributions of vertices and propagators, as it has been shown above.

Another regularization scheme for $\mathcal{N} = 1$ supersymmetric Yang–Mills theory exists which is the higher derivatives regularization scheme [11, 10]. According to Ref. [11], $\mathcal{N} = 2$ supersymmetry can be maintained by the regulator piece of the Lagrangian in HDR. The scheme is discussed in detail in Ref. [4] for the nonsupersymmetric case. A direct supersymmetric generalization of the regularization by higher derivatives has been constructed in Ref. [10]. This generalization has been considered in detail also in the paper [11], in particular in the background field technique. As has been explicitly shown in Refs. [11, 10], at one loop level HDR regularizes all the supergraphs in a gauge invariant manner and this repeats the corresponding construction in the nonsupersymmetric version of HDR [4]. However, when applied to explicit examples, this approach is known to yield incorrect results in Landau gauge [42]. A number of proposals have been put forward to treat this problem [43, 44]. As was shown in Ref. [44], the contradiction, noticed in [42] is related to the singular character of Landau gauge. In all other covariant gauges the method works and to include also the Landau gauge one has to add one more Pauli-Villars field to get the correct result [43, 44]. Having used this regularization, new scheme has been proposed in Refs. [45, 46, 47]. Calculations in the higher derivative regularization in terms of superfields can be found in Refs.[48, 49, 50, 45, 46, 47]. To perform the calculation, it is proposed to break the gauge symmetry by using some HDR scheme with usual derivatives instead of covariant derivatives and then to restore the ST identity for the effective action by using some noninvariant counterterms. This problem has been solved in Refs. [45, 46, 47]. All the arguments, given above for the regularization by the dimensional reduction in favor of scale independence of the kernels of dressed mean superfields are valid also for the regularization by higher derivatives. The only difference is that to remove the regularization by higher derivatives we take the limit $\Lambda \to \infty$ instead of $\epsilon \to 0$ as it was for the case of dimensional reduction, where Λ is the regularization scale of HDR.

So far the pure gauge sector has been considered. Let us look now at the matter two point functions. Schematically, one can write the two point vertex as

$$\int d^8z d^8z' \bar{\Phi}(z) G_{\Phi}(z-z')\Phi(z').$$

The function $G_{\Phi}(z-z')$ can be divided into two equal parts $\tilde{G}_{\Phi}$,

$$\int d^8z' \tilde{G}_{\Phi}(z_1 - z')\tilde{G}_{\Phi}(z'-z_2) = G_{\Phi}(z_1 - z_2).$$

This is a product in the momentum space. Now we define the new fields $\tilde{\Phi}$,

$$\int d^8z' \tilde{G}_{\Phi}(z-z')\Phi(z') \equiv \tilde{\Phi}(z),$$

and represent the effective action in terms of these fields. In particular, the divergent part of the function $\tilde{G}_{\Phi}$ can be absorbed into the redefinition of the Yukawa couplings and

masses. However, in $\mathcal{N} = 4$ supersymmetric theory masses are absent and the Yukawa coupling (which coincides with the gauge coupling) is not renormalized due to the structure of the Yukawa terms in the classical action. Thus, in terms of the dressed effective fields $\tilde{\Phi}$ and $\tilde{V}$ the effective action does not have any dependence on the UV and IR scales for $\mathcal{N} = 4$ supersymmetric theory.

In conclusion, all the correlator Lcc with all the possible contributions included turns out to be totally finite in $\mathcal{N} = 4$ super Yang-Mills theory in the Landau gauge and this property can be used to find it exactly in all the orders of the perturbation theory. The vertex Lcc in spite of being scale independent cannot be found by conformal symmetry since the external auxiliary superfield L does not propagate (it is not in the measure of the path integral). Three point connected Green functions of supermultiplets containing physical fields (like vector supermultiplet or matter supermultiplet) could be fixed by conformal symmetry up to some coefficient depending on the gauge coupling and number of colours.

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